## Linear systems, row reduction and echelon forms: Part 2

We now illustrate the row reduction algorithm to find the reduced echelon form of any matrix and how this allows for the solving of linear systems.

**Example:** Solve the following linear system:

$$3x_2 - 6x_3 + 4x_4 = -5$$
$$3x_1 - 7x_2 + 8x_3 + 8x_4 = 9$$
$$3x_1 - 9x_2 + 12x_3 + 6x_4 = 15$$

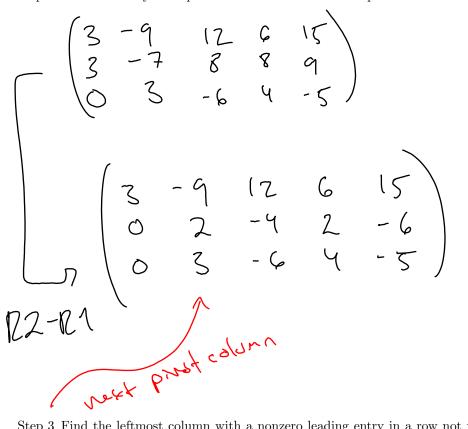
Step 0 Write the augmented matrix for the system.

$$\begin{pmatrix}
3 & -7 & 8 & 9 \\
3 & -7 & 8 & 9
\end{pmatrix}$$
3 -9 12 6 15

Step 1 Begin with the leftmost column (this is a pivot column). Select a nonzero entry in this column as a pivot and, if necessary, interchange rows to move this entry into the pivot position.

Prod 3 -9 12 6 15 3 -7 8 8 9 0 3 -6 4 -5

Step 2 Use elementary row operations to create zeros in all positions below the pivot.



Step 3 Find the leftmost column with a nonzero leading entry in a row not yet containing a pivot. (This is the next pivot column.) Apply steps 1 and 2 to this column.

Step 4 Repeat steps 1 - 3 until there are no more nonzero rows to modify.

Step 5 Beginning with the rightmost pivot and working upward and to the left, use elementary row operations to create zeros above each pivot.

$$\begin{pmatrix}
3 & -9 & 12 & 6 & 15 \\
0 & 2 & -4 & 2 & -6
\end{pmatrix}$$

$$\begin{pmatrix}
3 & -9 & 12 & 0 & -9 \\
0 & 2 & 4 & 0 & -14
\end{pmatrix}$$

$$\begin{pmatrix}
3 & -9 & 12 & 0 & -9 \\
0 & 2 & 4 & 0 & -14
\end{pmatrix}$$

$$\begin{pmatrix}
3 & -9 & 12 & 0 & -9 \\
0 & 1 & 2 & 0 & -9
\end{pmatrix}$$

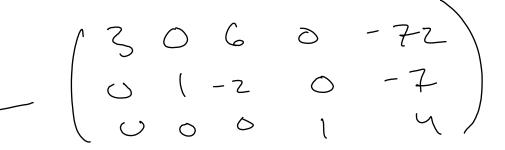
$$\begin{pmatrix}
3 & -9 & 12 & 0 & -9 \\
0 & 1 & -2 & 0 & -7
\end{pmatrix}$$

$$\begin{pmatrix}
3 & -9 & 12 & 0 & -9 \\
0 & 1 & -2 & 0 & -7
\end{pmatrix}$$

$$\begin{pmatrix}
3 & -9 & 12 & 0 & -9 \\
0 & 1 & -2 & 0 & -7
\end{pmatrix}$$

$$\begin{pmatrix}
3 & 0 & 6 & 0 & -72 \\
0 & 0 & 0 & 1 & 4
\end{pmatrix}$$

$$\begin{pmatrix}
41 + 9R2 & 3 & 0 & 6 & 0 & -72 \\
0 & 0 & 0 & 1 & 4
\end{pmatrix}$$



Step 6 Use scaling operation to make each pivot 1 and write down the corresponding system of equations.

$$\frac{1}{3}21 \left( 1020 - 24 \right) \\ 01 - 20 - 4 \right) \\ x_1 + 2x_3 = -24 \\ x_2 - 2x_3 = -7 \\ x_4 = 4$$

Step 7 Identify free and basic variables and give a parametric description of the solution set.